

## LRSD.WK1 - LAKE/RESERVOIR SAMPLING DESIGN WORKSHEET

Version 1.0 - December 1988

William W. Walker, Jr., Environmental Engineer, 1127 Lowell Road, Concord, MA 01742

LRSD.WK1 is a Lotus-123 worksheet which has been created to facilitate statistical evaluation of lake and reservoir sampling program designs. The assumed objective of the monitoring program is to estimate the longterm geometric mean concentration at a given station and/or to detect a step change in the longterm mean between two periods of monitoring. Samples would normally be taken from the epilimnion during the growing season for characterization of trophic state. The precision of the geometric mean is slightly higher than the precision of the arithmetic mean for variables which are lognormally distributed. For purposes of survey design, however, the distinction between the two is usually negligible (i.e., the coefficient of variation (CV) of the geometric mean ~ the CV of the arithmetic mean). The worksheet employs a modified version of the methodology described by Smeltzer et al. (1988) for estimating the precision of longterm means calculated from lake survey data.

The sampling program design is specified by the number of years of baseline monitoring, season length (days per year), and sampling interval (days between samples, e.g., 7 for weekly sampling). Precision in the longterm geometric mean is calculated from within-year and among-year variance components (Walker, 1980, Knowlton et al., 1984). Variance components, expressed as standard deviations on a base-e logarithmic scale, can be estimated from prior monitoring data for a particular station and water quality component using a one-way analysis of variance.

Otherwise, literature values may be used for these parameters, as summarized by Smeltzer et al. (1988) for various lake and reservoir data sets (see APPENDIX).

The effects of serial correlation (date-to-date within a given year) on the precision of yearly and longterm means are considered using the "effective sample size" concept (Matalas and Langbein, 1962; Lettenmaier, 1976). Experience with several lake data sets suggests that autocorrelation can become important at high monitoring frequencies (e.g., weekly or more frequent). Autocorrelation reduces the effective number of samples for calculating the yearly mean. The program requires an estimate of the serial correlation coefficient for a 1-day sampling frequency. Values in the range of .78 to .87 were estimated by Lettenmaier (1976) from 7 intensive data sets. Year-to-year variations in the mean are assumed to be serially independent.

Equations used for calculating the variance of and confidence factors for the geometric mean calculated for a given set of variance components, autocorrelation coefficient, and survey design are given below:

$$S_m^2 = S_y^2/N_y + S_d^2/N_y N_{de} \quad = \text{variance of longterm log}_e\text{-mean}$$

$$N_{de} = \text{FUNCTION}(N_d, r, k) \quad (\text{Matalas and Langbein, 1962})$$

$$CV = [S_m^2]^{1/2} \quad = \text{coefficient of variation of geometric mean}$$

$$f_l = \exp(-t S_m) \quad = \text{lower confidence factor for geometric mean}$$

prob [ ( true mean / estimated mean) >  $f_l$  ] ~ 95%

$$f_u = \exp(t S_m) \quad = \text{upper confidence factor for geometric mean}$$

prob [ ( true mean / estimated mean) <  $f_u$  ] ~ 95%

where,

$k$  = sampling interval (days between samples)

$t$  = t statistic with  $N_y N_{de} - 1$  degrees of freedom, area of each tail = 5%

$S_y$  = among-year standard deviation  $N_y$  = number of monitoring years

$S_d$  = within-year standard deviation  $N_d$  = number of sampling dates per year

$N_{de}$  = effective sampling dates per year  $r$  = lag 1-day autocorrelation coefficient

The t-test (Montgomery and Loftis, 1987) is employed to test for a significant difference in the longterm geometric mean calculated using data from two separate time periods. The test is applied to log-transformed data and the null hypothesis is that the means of the logarithms are not significantly different:

$$t = (m_2 - m_1) / S_{2-1}$$

$$S_{2-1} = (S_{m,1}^2 + S_{m,2}^2)^{1/2}$$

$$\text{dof} = N_{y,1} N_{de,1} + N_{y,2} N_{de,2} - 2$$

**Null Hypothesis** :  $m_1 = m_2$  is accepted if  $|t| < t_{\alpha, \text{dof}}$

where,

1,2 = subscripts denoting first and second time periods, respectively

$S_{2-1}$  = standard error of difference in log means between periods 1 and 2

$m_i$  = log-mean for period  $i$                       dof = degrees of freedom

$\alpha$  = significance level

**LRSD.WK1** estimates two statistics relevant to detection of a step change with a t-test:

(1) "**Minimum Detectable Change (MDC%)**" is defined by Spooner et al. (1987):

$$t = t_{\alpha, \text{dof}}$$

$$|m_2 - m_1| = t_{\alpha, \text{dof}} S_{2-1}$$

$$\text{MDC\%} = 100 [1 - \exp(-|m_2 - m_1|)] = 100 [1 - \exp(-t_{\alpha, \text{dof}} S_{2-1})]$$

The **MDC%** equals the minimum estimated percent change in the geometric mean which could cause rejection of the null hypothesis, given the error variances of the log-means calculated for lake sampling frequencies during each time period.

(2) The "**Power%**" of the t-test is computed using equations derived from Lettenmaier (1976):

$$N_t = \log_e(1 + C\%/100) / S_{2-1} \quad = \text{dimensionless trend number}$$

$$\text{Power\%} = 100 F(N_t - t_{\alpha, \text{dof}}, \text{dof})$$

where,

$C\%$  = hypothetical step change in geometric mean (%)

$F$  = cumulative frequency distribution of  $t$

This statistic equals the probability of detecting a specified percent change in the geometric mean (i.e., probability that null hypothesis would be rejected if the specified change of magnitude **C%** actually occurred), given the error variances of the log-means calculated from lake sampling frequencies during each time period.

Both of **MDC%** and **Power%** statistics are sensitive to sampling interval and duration. The specified within-year and among-year variance components are assumed to apply to both time periods. The assumed significance level ( $\alpha$ ) for both statistics is 5% for a one-tailed t-test (~appropriate for detecting a change in a known direction) and 10% for a two-tailed t-test (~appropriate for detecting a change in an unknown direction). If this is confusing, welcome

to the club.

Worksheet organization is illustrated in Table 1. Each column represents a separate case. This facilitates comparison of alternative sampling program designs. The original worksheet permits evaluation of six cases (columns) simultaneously. Additional columns may be added as required, using the Lotus copy command (make sure to copy entire column, rows 1-430).

The following information is entered by the user for each case or column:

<b>Case Label</b>	for labeling graphs
<b>Within-Year Ln Std Deviation</b>	estimated from lake data and/or literature
<b>Among-Year Ln Std Deviation</b>	"
<b>Lag 1-Day Auto-Correlation Coef.</b>	"
<b>Number of Years (N)</b>	duration of baseline monitoring
<b>Sampling Duration</b>	days per year, e.g., growing season length
<b>Sampling Interval</b>	days between samples within each year
<b>Hypoth. Change in Longterm Mean C%</b>	for power computations

Program outputs specific for each column include:

**POWER %** = Probability of detecting a C% change which occurs in the longterm geometric mean, given N years of monitoring before and after the change

**MDC%** = Minimum detectable change in longterm geometric mean, given N years of sampling before the change and N years of sampling after the change

**CV(Longterm Mean)** = Expected coefficient of variation of longterm geometric mean computed from N years of data

**CV(Yearly Mean)** = Expected coefficient of variation of the geometric mean for each year of data

**95% Confidence Factors - Low & High** = Lower and upper 95% confidence limits for ratio of true geometric mean to measured geometric mean ( $f_l$  and  $f_u$  above)

**Sample Saturation %** = Effective sample size per year / maximum possible sample size, based upon autocorrelation effects (Lettenmaier, 1976)

Sensitivity analysis tables include:

**mdc% vs. years of monitoring for N years of baseline data**

minimum detectable change in longterm geometric mean for a fixed number years of baseline data (N) and variable years of post-baseline data (1 to 100)

**cv (longterm mean) % vs. years of monitoring**

coefficient of variation of longterm geometric mean for variable number of years of monitoring (1 to 100)

**power % vs. step change % for N years of monitoring before and after**

probability of detecting step changes in the range of 10 to 100% based upon N years of monitoring before the change and N years after the change

**power % vs. years of monitoring for N yrs of baseline data and change C%**

probability of detecting a fixed step change of C% based upon N years of data before the change and variable number of years (1-100) after the change.

Graphic outputs include 5 named graphs, as illustrated in Figures 1-5. To display each graph in sequence, invoke the '\g' macro by pressing 'ALT' and 'g' simultaneously. Because of a Lotus-123 quirk, only portions of the graph legends (range labels) appear on the printed figures; screen images are complete.

The example shown in Table 1 and Figures 1-5 illustrates sensitivity to sampling interval (cases = annual, bimonthly, monthly, biweekly, weekly, semiweekly) using variance components which are typical for total phosphorus and a 3-year baseline monitoring period (N).

## REFERENCES

Knowlton, M.F., M.V. Hoyer, J.R. Jones, "Sources of Variability in Phosphorus and Chlorophyll

and Their Effects on Use of Lake Survey Data, Water Resources Bulletin, Volume 20, pp. 397-407, 1984.

Lettenmaier, D.P., "Detection of Trends in Water Quality Data From Records with Dependent Observations", Water Resources Research, Vol. 12, No. 5, pp. 1037-1046, October 1976.

Matalas, N.C. and W.B. Langbein, "Information Content of the Mean", Journal of Geophysical Research, Vol. 67, No. 9, pp. 3441-3448, 1962.

Montgomery, R.H. and J.C. Loftis, "Applicability of the t-Test for Detecting Trends in Water Quality Variables", Water Resources Bulletin, Vol. 23, No. 4, pp. 653-662, August 1987.

Smeltzer, E., W.W. Walker, and V. Garrison, "Eleven Years of Lake Eutrophication Monitoring in Vermont: A Critical Evaluation", presented at National Conference on Enhancing State Lake Management Programs, U.S. Environmental Protection Agency, Chicago, Illinois, May 12-13, 1988.

Snedecor G.W. and W.G. Cochran, Statistical Methods, Iowa State University Press, Ames, Sixth Edition, 1967.

Spooner, J. C.J. Jamieson, R.P. Maas, and M.D. Smolen, "Determining Statistically Significant Changes in Water Pollutant Concentrations", Lake and Reservoir Management, Volume III, North American Lake Management Society, pp. 195-201, 1987.

Walker, W.W., Jr., "Analysis of Water Quality Variations in Reservoirs: Implications for Monitoring and Modeling Efforts", in Stefan. H.G., ed., Surface Water Impoundments, American Society of Civil Engineers, New York, June 1980.

**Table 1**  
**LRSD Worksheet**

**LAKE/RESERVOIR SAMPLING DESIGN      LRSD-1.0      W. WALKER      DEC 1988**

Press 'ALT-G' for graphs

<b>INPUTS.....</b>	<b>SENSITIVITY TO SAMPLING INTERVAL - TOTAL P</b>					
case labels ----->	<b>ANNUAL</b>	<b>BIMONTHLY</b>	<b>MONTHLY</b>	<b>BIWEEKLY</b>	<b>WEEKLY</b>	<b>SEMIWEEKLY</b>
among-year ln std dev	0.12	0.12	0.12	0.12	0.12	0.12
within-year ln std dev	0.3	0.3	0.3	0.3	0.3	0.3
lag 1-day auto-correlation	0.8	0.8	0.8	0.8	0.8	0.8
sampling duration = N (yrs)	4	4	4	4	4	4
sampling season (days/year)	180	180	180	180	180	180
sampling interval (days)	180	60	30	14	7	4
hypothet. step change C (%)	25	25	25	25	25	25

<b>OUTPUTS.....</b>						
power for detecting C %	18.7	41.1	56.6	67.6	72.0	73.2
minimum detectable change %	35.9	22.6	18.4	16.0	15.1	14.9
cv (longterm geom. mean ) %	16.2	10.5	8.6	7.4	7.0	6.9
cv ( yearly geom. mean ) %	30.0	17.3	12.3	8.7	7.3	6.8
95% confid. factor - low	0.684	0.828	0.863	0.883	0.890	0.891
95% confid. factor - high	1.463	1.208	1.158	1.133	1.124	1.122
sample saturation %	4.9	14.6	29.1	57.6	83.0	93.6
total samples per season	1.0	3.0	6.0	12.9	25.7	45.0
total samples per N years	4.0	12.0	24.0	51.4	102.9	180.0

**mdc% vs. years of monitoring for N years of baseline data**

1	50.4	33.3	27.5	24.1	22.9	22.5
2	41.9	26.9	22.1	19.2	18.2	18.0
3	38.1	24.1	19.7	17.2	16.3	16.0
4	35.9	22.6	18.4	16.0	15.1	14.9
5	34.4	21.6	17.6	15.2	14.4	14.2
6	33.3	20.8	17.0	14.7	13.9	13.7
7	32.5	20.3	16.5	14.3	13.5	13.3
8	31.9	19.9	16.2	14.0	13.3	13.1
9	31.4	19.5	15.9	13.8	13.0	12.8
10	31.0	19.3	15.7	13.6	12.8	12.6
100	27.4	16.8	13.7	11.8	11.2	11.0

**cv (longterm geometric mean) % vs. years of monitoring**

1	32.3	21.1	17.2	14.8	14.0	13.8
2	22.8	14.9	12.1	10.5	9.9	9.8
3	18.7	12.2	9.9	8.6	8.1	8.0
4	16.2	10.5	8.6	7.4	7.0	6.9
5	14.4	9.4	7.7	6.6	6.3	6.2
6	13.2	8.6	7.0	6.1	5.7	5.6
7	12.2	8.0	6.5	5.6	5.3	5.2
8	11.4	7.4	6.1	5.2	5.0	4.9
9	10.8	7.0	5.7	4.9	4.7	4.6
10	10.2	6.7	5.4	4.7	4.4	4.4
100	3.2	2.1	1.7	1.5	1.4	1.4

**power % vs. step change % for N years of monitoring before and after**

10	8.8	14.7	19.0	23.0	25.2	25.3
20	14.8	31.6	42.8	53.2	57.5	58.7
30	23.1	51.8	68.2	79.6	83.7	84.8
40	32.8	70.0	86.1	93.9	96.0	96.4
50	43.3	83.6	95.0	98.6	99.2	99.3
60	54.5	91.9	98.4	99.7	99.9	99.9
70	64.2	96.2	99.5	99.9	100.0	100.0
80	72.2	98.3	99.8	100.0	100.0	100.0
90	78.8	99.2	99.9	100.0	100.0	100.0
100	84.1	99.6	100.0	100.0	100.0	100.0

**power % vs. years of monitoring for N yrs of baseline data and change C%**

1	9.0	20.6	29.6	37.1	40.4	41.5
2	12.6	28.7	41.3	52.5	57.3	58.7
3	15.9	34.1	49.3	61.6	66.2	67.5
4	18.7	37.8	55.0	67.1	71.8	73.2
5	21.0	40.7	59.0	71.0	75.7	77.1
6	22.9	43.0	61.7	73.9	78.6	79.9
7	24.4	45.0	63.8	76.0	80.7	82.0
8	25.7	46.7	65.5	77.8	82.4	83.6
9	26.8	48.0	66.9	79.2	83.6	84.9
10	27.8	49.0	68.0	80.3	84.7	85.9
100	37.9	60.4	79.1	89.7	92.8	93.6

**Figure 1****Named Graph: BAR**

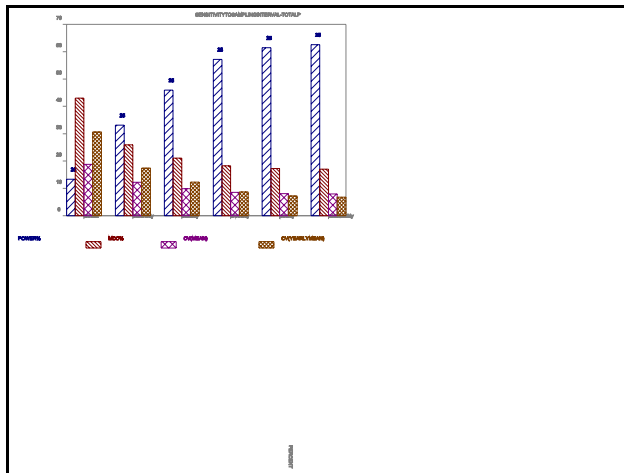
This graph shows the following values for each of six working columns in the spreadsheet:

POWER% = probability of detecting a change of C% based upon N years of monitoring before and after change.

MDC % = minimum detectable change in longterm mean for N years of monitoring before and after change.

CV(MEAN) = coefficient of variation of longterm geometric mean based upon N years of monitoring

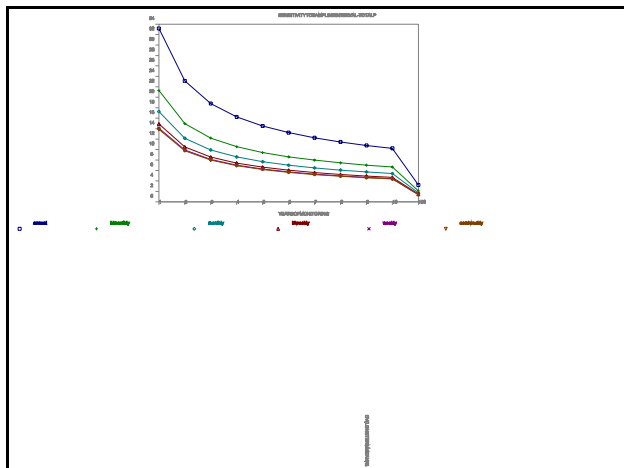
CV(YEARLY MEAN) = coefficient of variation of geometric mean for individual year

**Figure 2****Named Graph: CVMEAN**

X-Axis = X years of monitoring

Y-Axis = coefficient of variation of longterm geometric mean calculated from X years of data

Each line represents a separate column in the worksheet.

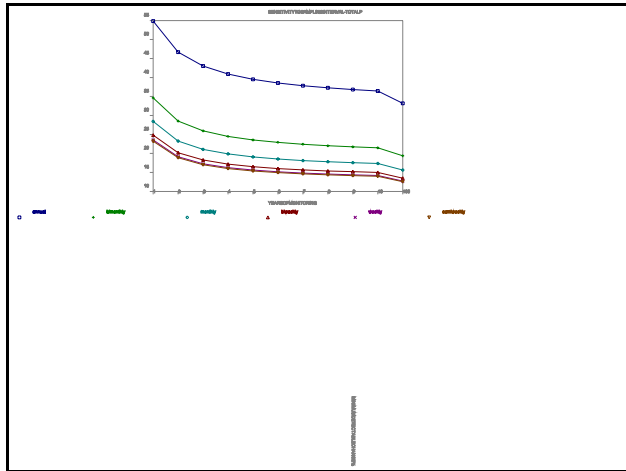


**Figure 3****Named Graph: MDC**

X-Axis = X years of monitoring after  
N years of baseline monitoring

Y-Axis = minimum detectable change  
in longterm mean, based upon  
comparison of N years of baseline  
data with X years of data collected  
after the change.

Each line represents a separate  
column in the worksheet.

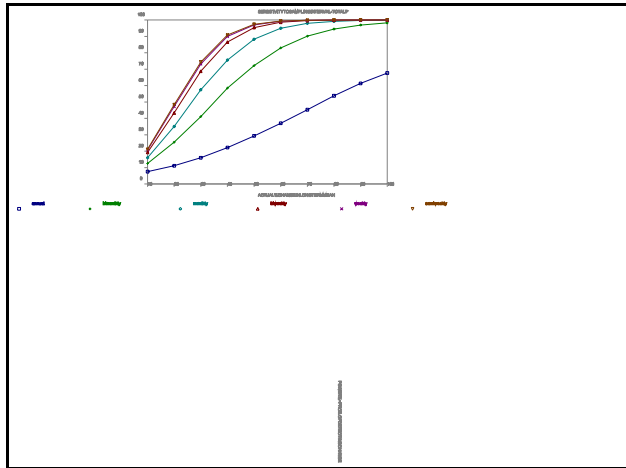


**Figure 4**  
**Named Graph: POWER**

X-Axis = Actual Change in Longterm Mean (%)

Y-Axis = Probability of Detecting Change, based upon N years of monitoring before change and N years of monitoring after change.

Each line represents a separate column in the worksheet.

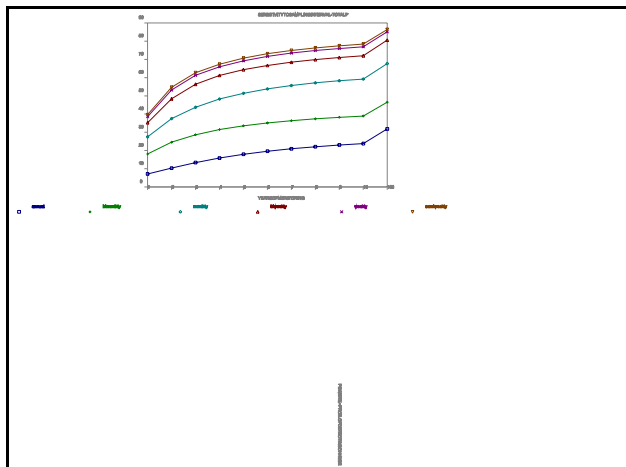


**Figure 5**  
**Named Graph: POWER2**

X-Axis = X Years of Monitoring after N years of Baseline Monitoring

Y-Axis = Probability of detecting a fixed percent change (C%), based upon comparison of N years of baseline data with X years of data collected after the change.

Each line represents a separate column in the worksheet.



## APPENDIX- LRSD 1.0

**Computation of Variance Components from Lake Survey Data (Modified from Smeltzer et al, 1988)**

The following procedure is designed for application to data from one lake station monitored for  $N_y$  years at an average of  $N_d$  sampling dates per year, within appropriate depth and seasonal strata (e.g., mixed layer, summer or growing season). A total of 20 observations over a 3-year period is recommended as a minimal basis for estimating station-specific variance components to be used in survey design calculations; otherwise, greater weight should be given to literature values (see figures below from Smeltzer et al., 1988; also, Knowlton et al., 1984).

1. Calculate means (or medians) of samples by sampling date. If the sampling design includes at least three observations per date (e.g., replicates or multiple sample depths within the mixed layer), taking medians provides a degree of protection against errant observations.
2. Transform the daily summary values to natural logarithms. Set any "zero" values equal to the lower detection limit before transforming.

$N_t$  = total number of sampling dates

$N_y$  = total number of years

$n_i$  = number of observations for year  $i$

$N_d$  = average ( $n_i$ ) =  $N_t/N_y$

3. Conduct a one-way analysis of variance (Snedecor and Cochran, 1967) with groups defined based upon sampling year. The ANOVA yields the following mean square statistics:

$M_y^2$  = mean squared deviation among years

$M_d^2$  = mean squared deviation within years

4. Estimate among-year and within-year standard deviations:

$S_y = [ (M_y^2 - M_d^2) / N_d ]^{1/2}$  = year-to-year standard deviation of  $\ln(\text{conc})$

$N_0 = (N_t - \text{SUM}(n_i^2)/N_t) / (N_y - 1)$  = adjusted samples per year ( $\sim N_d$ )

$S_d = [ M_d^2 ]^{1/2}$  = date-to-date standard deviation of  $\ln(\text{conc})$

**APPENDIX- LRSD 1.0**

**Lake Variance Component Distributions - Smeltzer et al.(1988)**